Nonisothermal Elastoviscoplastic Snap-Through and Creep Buckling of Shallow Arches

R. Riff*
Georgia Institute of Technology, Atlanta, Georgia

The problem of buckling of shallow arches under transient thermomechanical loads is investigated. The analysis is based on nonlinear geometric and constitutive relations and is expressed in a rate form. The material constitutive equations are capable of reproducing all nonisothermal, elastoviscoplastic characteristics. The solution scheme is capable of predicting response that includes pre- and postbuckling with creep and plastic effects. The solution procedure is demonstrated through several examples that include both creep and snap-through behavior.

Introduction

ELASTIC snap-through of low arches under quasistatic loads has been the subject of several investigations. The significance of the problem, insofar as it illustrates certain important features in more complicated buckling problems of plastes and shells, was pointed out by Marguerre,[1] who constructed a simplified mechanical model to demonstrate these features. Timoshenko[2] obtained an approximate solution to the problem of a low arch under a uniformly distributed load. Biezeno[3] considered the problem of a low-parabolic arch loaded laterally at the midpoint with a concentrated load. He also investigated snap-through buckling of a shallow spherical cap, pinned along its circular boundary, under the action of a concentrated load applied along the axis of rotational symmetry. He presented his approximate solutions by means of load-deflection curves and equations from which the critical load could be calculated.

In 1952, Fung and Kaplan[4] investigated the problem of low-pinned arches of various initial shapes and spatial distributions of the lateral load. Their results show that a very shallow arch snap through symmetrically, whereas a higher arch buckles asymmetrically. They also ran a limited number of experimental tests, and their experimental data are in good agreement with their theoretical results. About the same time, Hoff and Bruce[5], in investigating the possibility of snap-through buckling of a low-pinned arch with a half-sine-wave initial shape under a half-sine-wave distributed dynamic load (suddenly applied with infinite duration), obtained results for the quasistatic load case that are identical to those obtained by Fung and Kaplan for the same problem.

In 1962, Gjelsvik and Bodner[6] obtained an approximate solution to the problem of a low-circular arch with a concentrated load at the midpoint of the arch and clamped boundary conditions. They also reported on experimental results. Schreyer and Masur[7] obtained an exact solution to their problem (and for the load case uniform pressure), and they showed that for the concentrated load case, the arch snaps through symmetrically regardless of the value of the rise parameter. Masur and Lo[8] presented a general discussion of the behavior of the shallow circular arch regarding buckling, postbuckling, and imperfection sensitivity. Snapping of low-pinned arches resting on an elastic foundation has been investigated by Simitses.[9] This system exhibits all forms of experimentally observed buckling phenomena (smooth and violent) and of theoretically predicted responses (limit point, bifurcation with stable branching, and bifurcation with unstable branching), and it is presented with sufficient detail in Ref. 10. Experimental results have also been reported by Roorda.[11]

The effects of inelastic material behavior found their way into the literature since the 1960's. Onat and Shu[12] considered the behavior to be one of rigid-perfectly plastic. Fromciosi, Augusti, and Sparacio[13] discussed the collapse of arches under repeated loads with inelastic material behavior. Studies of inelastic snap-through buckling of shallow arches also were reported by Lee and Murphy.[14] In addition, Augusti[15] investigated plastic buckling of a model of a three-hinged arch in 1968, and a more complete analysis of the same model was provided by Batterman[16] in 1971. Finally, the reader who is interested in the ultimate strength of parabolic steel arches with bracing system is referred to Komatsu,[17] who considers inelastic in-plane and out-of-plane instabilities and provides design formula for each case.

Creep buckling of shallow arches has been investigated by Huang and Nachbar,[18] Krajcinovic,[19] and Botros and Bienek.[20] The elastic response of arches under sudden (dynamic) application of the external loads has been reported by Hoff and Bruce,[5] Hsu,[21,22] and Lock.[23] For a more complete bibliography see Ref. 24. As far as the authors know, no work has been reported on the nonisothermal elastoviscoplastic behavior of shallow arches. The purpose of this paper is to demonstrate the effect of highly nonlinear material behavior on the snap through and creep buckling of shallow arches.

Elastothermoviscoplastic Constitutive Relations

The prediction of buckling loads and postbuckling behavior of structural components, like shallow arches, made of a realistic metallic material and subjected to nonisothermal thermomechanical loads has increased in importance in recent years.

Under this kind of severe loading conditions, the structural behavior is highly nonlinear due to the combined action of geometrical and material nonlinearities. On one side, finite deformation in a stressed structure introduces nonlinear geometric effects. On the other side, physical nonlinearities arise even in small strain regimes, whereby inelastic phenomena play a particularly important role. From a theoretical standpoint, nonlinear constitutive equations should be applied only in connection with nonlinear transformation measures (imply-

Presented as Paper 87-0806 at the AIAA/ASME/ASCE/AHS 28th Structures, Structural Dynamics, and Material Conference, Monterey, CA., April 6–8, 1987; received 27, 1987; revision received June 3, 1988.

*Assistant Professor of Aerospace Engineering. Member AIAA.

ing both deformation and rotations). However, in almost all of the works in this area,[25] the two identified sources of nonlinearities are always separated. The separation yields, at one end of the spectrum, problems of large response, whereas at the other end problems of viscous and/or nonisothermal behavior in the presence of small strain.

The classical theories in which the material response is characterized as a combination of distinct elastic, thermal, time-independent inelastic (plastic) and time-dependent inelastic (creep) deformation components cannot explain some phenomena that can be observed in complex thermomechanical loading histories. This is particularly true when high-temperature nonisothermal processes must be taken into account. There is a sizeable body of literature[25,26] on phenomenological constitutive equations for the rate and temperature-dependent plastic deformation behavior of metallic materials. However, almost all of these new "unified" theories are based on small strain theories, and several suffer from some thermodynamic inconsistencies.

In a previous paper,[27] the authors have presented a complete set of constitutive relations for nonisothermal, large strain, elastoviscoplastic behavior of metals. It was shown there[27] that the metric tensor in the convected (material) coordinate system can be linearly decomposed into elastic and (visco) plastic parts. So a yield function was assumed, which is dependent on the rate of change of stress on the metric, on the temperature, and on a set of internal variables. Moreover, a hypoelastic law was chosen to describe the thermoelastic part of the deformation.

A time- and temperature-dependent viscoplasticity model was formulated in this convected material system to account for finite strains and rotations. The history and temperature dependence were incorporated through the introduction of internal variables. The choice of these variables and their evolution were motivated by thermodynamic considerations.

The nonisothermal elastoviscoplastic deformation process was described completely by "thermodynamic state" equations. Most investigators[25,26] (in the area of viscoplasticity) employ plastic strains as state variables. The authors' previous study[27] shows that, in general, use of plastic strains as state variables may lead to inconsistencies with regard to thermodynamic considerations. Furthermore, the approach and formulation employed in previous works lead to the condition that all of the plastic work is completely dissipated. This, however, is in contradiction with experimental evidence, from which it emerges that part of the plastic work is used for producing residual stresses in the lattice that, when phenomenologically considered, causes hardening. Both limiations were excluded from this[27] formulation. Accuracy of the formulation was checked on a wide range of examples.[28]

The constitutive relation will be rephrased here in some different form. For brevity we compile only some notations and fundamental relations that are used in the formulation of the intended constitutive law. For details see Refs. 27 and 28.

Concerning the formulation of constitutive laws, it is advantageous to use a material (convected) coordinate system. The transformation from the underformed state (metric $\overset{\circ}{g}_{ik}$) to the deformed state g_{ik} can be represented by the tensor

$$f^i_k = \overset{\circ}{g}{}^{ir}\, g_{rk} \tag{1a}$$

or

$$(f^{-1})^i_k = g^{ir}\, \overset{\circ}{g}_{rk} \tag{1b}$$

The total deformation rate is defined by

$$d^i_k = \tfrac{1}{2} g^{ir}\dot{g}_{rk} = -\tfrac{1}{2} g_{ir}\dot{g}^{rk} = \tfrac{1}{2}(f^{-1})^i_r(\dot{f})^r_{.k} = -\tfrac{1}{2}(\dot{f}^{-1})^i_{.r} f^r_k \tag{2}$$

Here ($\dot{}$) denotes time material derivative. The rate of change of the metric tensor is given by

$$\dot{g}_{ik} = v_{i,k} + v_{k,i} \tag{3}$$

and $v_{i,k}$ are the material velocities gradients. Hence,

$$d_{ik} = \tfrac{1}{2}(v_{i,k} + v_{k,i}) \tag{4}$$

The expression

$$\overset{\nabla}{f}{}^i_k = (\dot{f})^i_{.k} + d^i_k f^r_r - d^r_k f^i_r = \mathrm{sym}[(\dot{f})^i_{.k}] \tag{5}$$

represents the symmetric part of $(\dot{f})^i_{.k}$ or the covariant derivative with respect to time, also called the convected derivation, which is due to Zaremba and Jaumann.

The total deformation can be decomposed according to

$$f^i_k = \overset{\circ}{g}{}^{im} \overset{*}{g}_{mr}\, g_{sk} = \overset{(i)}{f}{}^i_r \overset{(r)}{f}{}^r_k \tag{6}$$

where the superscript (*) relates to a fictitious configuration defined by a fictitious reversible process with frozen internal variables. The decomposition of Eq. (6) leads to an additive decomposition of the deformation rate

$$d^i_k = \overset{(r)}{d}{}^i_k + \overset{(i)}{d}{}^i_k \tag{7}$$

$\overset{(r)}{d}{}^i_k$ is related to the reversible deformations, whereas $\overset{(i)}{d}{}^i_k$ denotes the remaining part of the deformation rate.

For the description of the stress state, we introduce the Kirchoff stress tensor s^i_k, which is connected with the real Cauchy stress tensor σ^i_k by the relation

$$s^i_k = (\overset{\circ}{\rho}/\rho)\sigma^i_k \tag{8}$$

Assuming that the elastic behavior is not affected essentially by inelastic deformations, the following hypoelastic incremental law was chosen[27]:

$$\overset{(r)}{d}{}^i_k = \frac{1}{2G}\overset{\nabla}{t}{}^i_k + \left[\frac{1}{9K}\dot{s}^r_r + \alpha\dot{T}\right]\delta^i_k \tag{9}$$

where t^i_k is the weighted stress deviator, G the shear modulus, K the bulk modulus, and α the coefficient of thermal expansion.

The following constitutive relations were established[27] for the inelastic behavior. Yield condition:

$$F = (\, t^i_k - c\overset{\circ}{\rho} g \beta^i_k)(t^k_i - c\overset{\circ}{\rho} g \beta^k_i) - k^2(A,T) = f^2 - k^2 > 0 \tag{10}$$

Accompanying equilibrium state:

$$\bar{F} = (\bar{t}^i_k - c\overset{\circ}{\rho} g \beta^i_k)(\bar{t}^k_i - c\overset{\circ}{\rho} g \beta^k_i) - k^2(A,T) = \bar{f}^2 - k^2 = 0 \tag{11}$$

Evolution law for inelastic deformations:

$$\overset{(i)}{d}{}^i_k = 2\dot{\lambda}(\bar{t}^i_k - c\overset{\circ}{\rho} g \beta^i_k) \tag{12}$$

with

$$\dot{\lambda} = \frac{1}{4\eta}\left(\sqrt{\frac{(t^i_k - c\overset{\circ}{\rho} g \beta^i_k)(t^k_i - c\overset{\circ}{\rho} g \beta^k_i)}{k^2}} - 1\right) \tag{13}$$

$$\bar{t}^i_k = \frac{1}{1 + 4\eta\dot{\lambda}}(t^i_k - c\overset{\circ}{\rho} g \beta^i_k) + c\overset{\circ}{\rho} g \beta^i_k \tag{14}$$

Evolution laws for the internal variables:

$$\dot{A} = \frac{1}{\overset{\circ}{\rho}}\bar{t}^i_k \overset{(i)}{d}{}^i_k \tag{15}$$

$$\overset{\nabla}{\beta}{}^i_k = \xi \overset{(i)}{d}{}^i_k \tag{16}$$

If

$$F = 0 \text{ and } \frac{\partial F}{\partial s_k^i} \overset{\nabla}{s}{}_k^i + \frac{\partial F}{\partial T}\dot{T} > 0 \tag{17}$$

then

$$d_k^i = \overset{(r)}{d}{}_k^i \tag{18}$$

$$\overset{(i)}{d}{}_k^i = 0 \text{ and } \overset{\nabla}{d}{}_k^i = 2\ddot{\lambda}(t_k^i - c\mathring{\rho}g\beta_k^i) \tag{19}$$

with

$$\ddot{\lambda} = \frac{1}{8\eta k^2}\left[2(t_k^i - c\mathring{\rho}g\beta_k^i)\,\overset{\nabla}{t}{}_i^k - \frac{\partial k^2}{\partial T}\dot{T}\right] \tag{20}$$

If

$$F = 0 \text{ and } \frac{\partial F}{\partial s_k^i} \overset{\nabla}{s}{}_k^i + \frac{\partial F}{\partial T}\dot{T} \leq 0 \tag{21}$$

or

$$F < 0 \tag{22}$$

$$d_k^i = \overset{(r)}{d}{}_k^i \tag{23a}$$

$$\dot{A} = 0 \tag{23b}$$

$$\overset{\nabla}{\beta}{}_k^i = 0 \tag{23c}$$

Within the developed frame, the elastoviscoplastic behavior is governed by the scalar material functions $c(s_k^i, T, A, \beta_k^i)$, $k^2(A,T)$, $g(s_k^i, R, \beta_k^i)$, $\xi(A,R,\beta_k^i)$, and $\eta(A,T,\beta_k^i)$. These material functions can be determined from a series of monotonic and cyclic processes with proportional and nonproportional paths at different temperature levels.[28]

General Formulation and Solution Schemes

The rate form of the constitutive equations suggests that a rate approach be taken toward the entire problem so that flow is viewed as history-dependent process rather than an event. For this purpose, a complete true ab-initio rate theory of kinematics and kinetics for continuum and curved thin structures, without any restriction on the magnitude of the transformation, was presented in Ref. 28. It is implemented with respect to a body-fixed system of convected coordinates, and it is valid for finite strains and finite rotations. The time dependence and large strain behavior are incorporated through the introduction of the time rates of change of the metric and of the curvature.

The constitutive law may be applied to the conservation of momentum via an appropriate variational principle. The principle of virtual power (or of virtual velocities) reads

$$\int_V \sigma^{ij}\delta v_{j,i}\,\mathrm{d}V - \int_V v\rho f^j \delta v_j\,\mathrm{d}V - \int_A \nu T^j \delta v_j\,\mathrm{d}A = 0 \tag{24}$$

where δv_j are the virtual velocities, f^j the body forces per unit mass, and νT^j the surface tractions. Total differentiation of Eq. (24) yields

$$\int_V \left(\frac{\mathrm{d}\sigma^{ij}}{\mathrm{d}t} + \sigma^{ij}\,\mathrm{d}_{.k}^k - v_{,k}^i\sigma^{kj}\right)\delta v_{j,i}\,\mathrm{d}V - \int_V \rho\frac{\mathrm{d}f^j}{\mathrm{d}t}\delta v_j\,\mathrm{d}V$$

$$- \int_A \nu\frac{\mathrm{d}T^j}{\mathrm{d}t}\delta v_j\,\mathrm{d}A + \int_V \sigma^{ij}\left(\frac{\mathrm{d}\delta v_j}{\mathrm{d}t}\right)_{,i}\mathrm{d}V - \int_V \rho f^i\frac{\mathrm{d}\delta v_j}{\mathrm{d}t}\mathrm{d}V$$

$$- \int_A \nu T^j\frac{\mathrm{d}\delta v_j}{\mathrm{d}t}\,\mathrm{d}A = 0 \tag{25}$$

At any instant Eq. (25) must be satisfied. The virtual velocity and its time derivative are then independent. Moreover, the last three terms of Eq. (25) are equivalent to Eq. (24). Hence, the principle of the rate of virtual power may be obtained in its concise form. For further classifiations, the total derivative of the stress components will be represented by the Jauman derivative, and the following integrals are defined by

$$I_e = \int_V \overset{\nabla}{\sigma}{}^{ij}\,\delta v_{j,i}\,\mathrm{d}V \tag{26}$$

$$I_d = \int_V (\sigma^{ij} d_k^k - \sigma^{kj} d_{.k}^i)\delta v_{j,i}\,\mathrm{d}V \tag{27}$$

$$I_r = \int_V \omega_{.k}^j \sigma^{kj}\delta v_{j,i}\,\mathrm{d}V \tag{28}$$

Then, substitution in Eqs. (23) yields the final form of the principle of the rate of virtual power:

$$I = I_e + I_d + I_r = \int_V \rho\frac{\mathrm{d}f^j}{\mathrm{d}t}\delta v_j\,\mathrm{d}V + \int_A \nu\frac{\mathrm{d}T^j}{\mathrm{d}t}\delta v_j\,\mathrm{d}A \tag{29}$$

The quasilinear nature of the principle of the rate of virtual power suggests the adoption of an incremental approach to numerical integration with respect to time. The availability of the field formulation provides assurance of the completeness of the incremental equations and allows the use of any convenient procedure for spatial integration over the domain V. In the present instance, the choice has been made in favor of a simple first-order expansion in time for the construction of incremental solutions from the results of finite-element spatial integration of the governing equations.

The procedure employed permits the rates of the field formulation to be interpreted as increments in the numerical solution. This is particularly convenient for the construction of incremental boundary condition histories.

The finite-element method for spatial discretization has been well documented (see, e.g., the books by Zienkiewicz[29] or Oden[30]) and will not be detailed here. The algebraic counterpart of Eq. (29) After the finite-element discretization (for detail see Ref. 28) is the well-known stiffness expression

$$[K]\{V\} = \{\dot{P}\} - \{\dot{F}\} \tag{30}$$

with the tangent stiffness matrix $[K]$, the vector of the incremental velocities $\{V\}$, and the vector out-of-balance force rates, external force rates $\{\dot{P}\}$ minus internal force rates $\{\dot{F}\}$. The homogenous case of Eq. (30) indicates either the nonuniqueness of the equilibrium path at a stable or unstable bifurcation point or the unique but unstable situation at a limit point. Hence, this criterion may be evaluated by a determinant check or supplementary eigenvalue analysis for the load parameter parallel to the loading process.

Even under the condition of static external loads and slowly growing creep effects, the presence of snap-through buckling makes the inertia effects significant. In dynamic analyses, the applied body forces include inertia forces. Assuming that the mass of the body considered is preserved, the mass matrix can be evaluated prior to the time integration using the initial configuration.

Finite-element solution of any boundary-value problem involves the solution of the equilibrium equation (global) together with the constitutive equation (local). Both equations are solved simultaneously in a step by step manner. The incremental form of the global and local equations can be achieved by taking the integration over the incremental time step $\Delta t = t_{j+1} - t_j$. The rectangular rule has been applied to execute the resulting time integration.

Clearly, the numerical solution involves iteration. A simplified version[31] of Riks Wempner constant-arch-length method

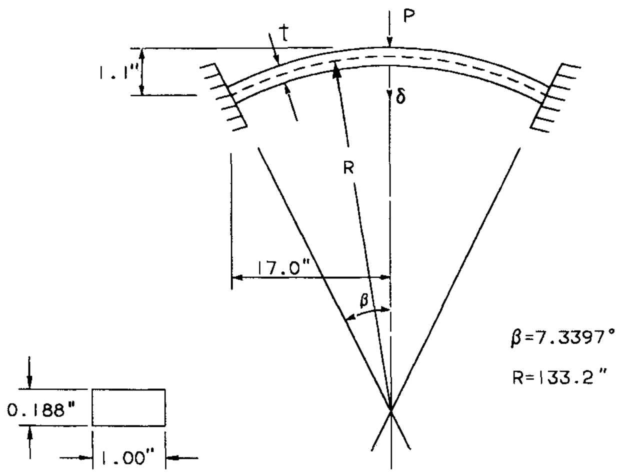
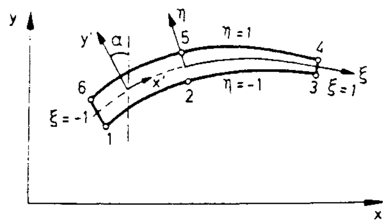


Fig. 1 Clamped circular arch.



$\xi_o = \xi . \xi_i$	
$\eta_o = \eta . \eta_i$	

Node	N_i
Corner $\xi_i = \pm 1, \eta_i = \pm 1$	$\frac{1}{4}(\xi_o + \xi^2)(1 + \eta_o)$
Midside $\xi_i = 0, \eta_i = \pm 1$	$\frac{1}{2}(1 - \xi^2)(1 + \eta_o)$

Fig. 2 Paralinear isoparametric element.

has been used. This iteration procedure, which is a generalization of the displacement control method, also allows to trace the nonlinear response beyond bifurcation points. In contrast to the conventional Newton-Raphson techniques, the iteration of the method takes place in the velocity and load rate space. The load step of the first solution in each increment is limited by controlling the length ds of the tangent. Either the length is kept constant in each step, or it is adapted to the characteristics of the solution. In each step the triangular-sized stiffness matrix has to be checked for negative diagonal terms, indicating that a critical point is reached.

Shallow Circular Clamped Arch

The theory and computational procedures described in the proceeding sections have been applied to the creep buckling analysis of a shallow circular clamped arch. The problem of the clamped arch, besides being of some practical interest, contains a number of similarities to that of the uniformly loaded spherical cap. The trend of results of the arch problem serves as a good indicator to the behavior of the latter.

The shallow circular clamped arch subjected to a single central concentrated load, as shown in Fig. 1, is analyzed. The material chosen for the numerical experimentation is the carbon steel C-45 (DIN 1720) with $E = 10^7$ psi, $\nu = 0.3$, and $\sigma_y = 2.7 \times 10^4$ psi at room temperature. The viscoplastic properties (the scalar material functions) were obtained in Ref. 28.

The analysis is performed with the aid of 24 paralinear isoparametric elements (Fig. 2). The paralinear isoparametric element is intended for the analysis of oriented structures where the geometry is such that the thickness is small compared to other dimensions. The characteristics of the element are defined by the geometry and interpolation functions, which are linear in the thickness direction and parabolic in the longitudinal direction (see Fig. 2). Consequently, the element

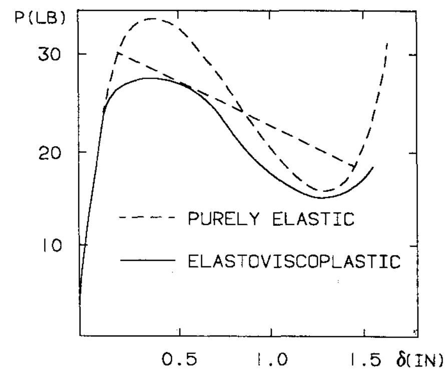


Fig. 3 Arch response.

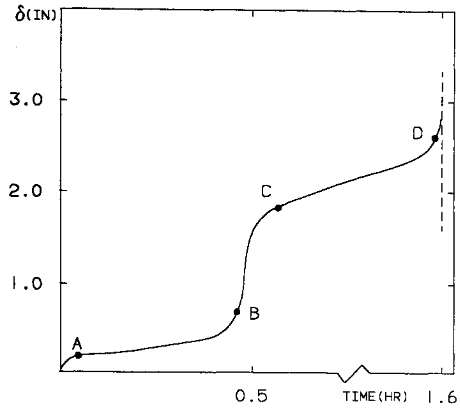


Fig. 4 Deflection time history.

allows for shear strain energy, since normals to a midsurface before deformation remain straight but not necessarily normal to the midsurface after deformation.

The elastic behavior, corresponding to both axisymmetric and asymmetric response, is shown on Fig. 3. These curves are in complete agreement with those produced by Gjelsvik and Bodner,[6] only because the Young's modulus and Poisson's ratio values used are virtually the same (carbon steel C-45 here and 2024-T4 aluminum alloy in Ref. 6). Note that these elastic response curves are hypothetical for our material but true for the 2024-T4 alloy. The true behavior for our material is elastoviscoplastic, and it is labeled as such on Fig. 3. Note that this curve represents quasistatic (steady-state) elastoviscoplastic response, as described by the chosen constitutive law. According to this, snapping occurs at a load of 26.20 lb, primarily because of the low-yield strength. Then, the postlimit point behavior seems to be primarily driven by viscoplastic behavior.

It is expected here that if loads up to approximately 14 lb are reached quasistatically and left applied for a long time, the primary response will be creep, and the critical creep collapse time will be extremely large. On the other hand, for loads between 14 and 26.2 lb (especially for the higher range), the behavior will be a combination of creep and snap-through buckling. This is best demonstrated by the curve on Fig. 4. The applied load is reached quasistatically in 13 min, and then it is kept constant. The curve of Fig. 4 depicts the behavior of the arch in terms of midpoint deflection vs time. Creep, initially, is very slow; then snap-through takes place in 32 min, curve BC, and then the creep behavior continues until a critical time to creep (creep buckling occurs) is reached after a total time 97 min. Note that for this loading condition, the critical time to creep in 97 min. Creep buckling and critical time to creep are based on the phenomenon that the deflection increased very rapidly. For loads higher than 26.2 lb, it is expected that snapping will occur early during quasistatic loading, and then the creep behavior will be similar to that shown on Fig. 4, past point c.

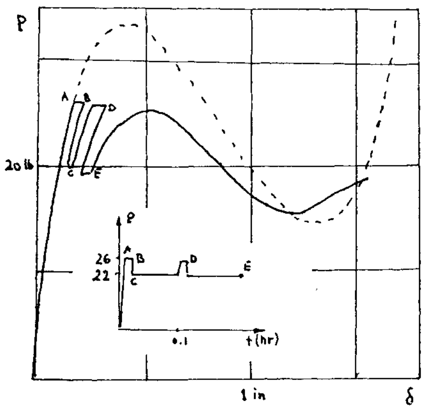


Fig. 5 Multicycle arch response.

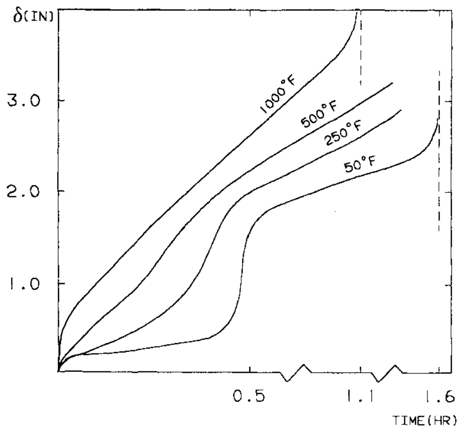


Fig. 6 Influence of temperature raise.

The next example considers the influence of cyclic loading on the response. Figure 5 illustrates the load deflection behavior of the arch under a cyclically applied external load. The load is increased, quasistatically, from zero to 26 lb in 5 min, then it is held constant for 2.5 min, after that it is reduced to 20 lbs and held constant for 50 min, then raised to 25.5 lb for 2.5 min, and finally it is reduced back to 20 lb held constant.

The steady-state response under this type of loading exhibits several relative maxima points, which may imply that snapping is imminent shortly after the load reaches the value of 26 lb (between points A and B on Fig. 5). The dashed curve corresponds to the hypothetical elastic static response, and it is only shown for comparison purposes.

The last example presented in Fig. 6 considers the influence of temperature on the arch behavior. The loading history is the same on the one shown on Fig. 4. The curve corresponding to $T = 50°F$ was discussed previously (Fig. 4), and it is used here as a basis for comparison. When the temperature is raised to 200°F (after this, the loading is applied), the time to snap is reduced to 26 min, whereas the critical creep collapse time is not appreciably affected. On the other hand, at the highest temperature $T = 1000°F$ for which results are shown. The critical creep collapse time is reduced to 62 min, and the steady-state response does not show a clear snap-through behavior. Different values of α were used for the different temperature in the elastic range.

Discussion

As noted earlier, the main thrust of this work has been to demonstrate the effect of highly nonlinear material behavior on the snap-through and creep buckling of shallow arches. It is evident that in the presence of both elastic and viscoplastic deformation the process of buckling assumes an entirely new character. Buckling develops as a time-temperature-dependent deformation process under constant or variable loads of magnitudes smaller than the elastic critical values. In arches under loads below the critical values the structure initially deforms quasistatically, with the thermoviscous terms manifesting themselves in the form of increasing displacement under, say, a constant load. When the magnitude of the displacements reaches a certain threshold state, the arch snaps dynamically into the postbuckling configuration and then continues quasistatic deformation again.

The material constitutive relation has been proven to be capable of reproducing the main characteristics of viscoplastic deformations. The modified Riks/Wempner iteration scheme has been found to be a versatile technique in the pre- and postcritical range.

The influence of thermomechanical coupling can become very large in stability problems. Such processes are always connected with a rapid growth of inhomogeneity of the state. Thorough investigation of such problems, however, must be performed with the necessary detail.

Acknowledgement

The research described above has been performed under NASA Grant No. 3-534. The financial support provided by NASA is gratefully acknowledged by the authors. The authors also wish to extend their thanks and appreciation to Dr. C. C. Chamis of the NASA-Lewis Research Center for his support and for the insights into the problem that he provided during our many technical discussions.

References

[1]Marrguerre, K., "Die Durchschlagskraft eines Schwach Gekrummten Balken," *Sitz. Berlin Math. Gess.*, Vol. 37, 1938, pp. 92–108.

[2]Timoshenko, S. P., "Buckling of Curved Bars with Small Curvature," *Journal of Applied Mechanics*, Vol. 2, No. 1, March 1935, pp. 17–23.

[3]Biezeno, C. B., "Das Durchschlagen eines Schwach Gekrummten Stabes," *Zeitschrift Ange. Math. und Mekh*, Vol. 18, 1938, pp. 21–29.

[4]Fung, Y. C. and Kaplan, A ., "Buckling of Low Arches or Curved Beams of Small Curvature," NACA TN-2840, 1952.

[5]Hoff, N. J. and Bruce, V. G., "Dynamic Analysis of the Buckling of Laterally Loaded Flat Arches," *Journal of Mathematics and Physics*, Vol. 32, No. 4, Jan. 1954, pp. 276–288.

[6]Gjelsvik, A. and Bodner, S. R., "The Energy Criterion and Snap Buckling of Arches," *J. Eng. Mech.*, Div. ASCE, Vol. 88, EM5, May 1962, pp. 87–108.

[7]Schreyer, H. L. and Masur, E. F., "Buckling of Shallow Arches," *J. Eng. Mech.*, Div. ASCE, Vol. 92, EM4, April 1966, pp. 1–17.

[8]Masur, E. F. and Lo, D. L. C., "The Shallow Arch-General Buckling, Post Buckling, and Imperfection Analysis," *Journal of Structural Mechanics*, Vol. 1, No. 1, 1972, pp. 91–107.

[9]Simitses, G. J., "Snapping of Low Pinned Arches on an Elastic Foundation," *Journal of Applied Mechanics*, Vol. 40, No. 3, Sept. 1973, pp. 741–774.

[10]Simitses, G. J., *Elastic Stability of Structures*, Prentice-Hall, Englewood Cliffs, NJ, 1976; second printing, R. E. Kreiger Publishing Co., Melbourne, FL, 1986.

[11]Roorda, J., "Stability of Structures with Small Imperfections," *Journal of Engineering Mechanics Division*, ASCE, Vol. 91, EM1, 1965, pp. 87–108.

[12]Onat, E. T. and Shu, L. S., "Finite Deformation of a Rigid Perfectly Plastic Arch," *Journal of Applied Mechanics*, Vol. 29. No. 3, 1962, pp. 549–553.

[13]Franciosi, V., Augusti, G., and Sparacio, R., "Collapse of Arches Under Repeated Loading," *Journal of the Structural Division*, ASCE, Vol. 90, ST1, Jan. 1964, pp. 165–179.

[14]Lee, L. H. N. and Murphy, L. M., "Inelastic Buckling of Shallow Arches," *Journal of Engineering Mechanics Division*, ASCE, Vol. 94, EM1, Jan. 1968, pp. 225–239.

[15]Augusti, G., "Buckling of Inelastic Arches: Simple Model," *Meccanica*, Vol. 3, 1968, pp. 102–105.

[16]Batterman, S. C. "Plastic Stability of Arches: Re-Consideration of a Model," *Israel Journal of Technology*, Vol. 9, 1971, pp. 467–476.

[17]Kamatsu, S., "Braced Steel Arches," *Steel Framed Structures: Stability and Strength*, edited by R. Narayanan, Elsevier Applied Science Publishers Ltd, London, 1985, Chap. 9.

[18]Huang, N. C. and Nachbar, W., "Dynamic Snap-Through of Imperfect Viscoelastic Shallow Arches," Univ. of California at La-Jolla, LaJolla, CA, TR-AF-AF0SR 1226-67, March 1967.

[19]Krajcinovic, D., "Creep Buckling of a Tied Arch," *Stability of Structures Under Static and Dynamics Loads,* American Society of Chemical Engineers, New York, 1977.

[20]Botros, F. R. and Bienek, M. P., "Creep Buckling of Structures," *Proceedings of the AIAA/ASME/ASCE/AHS 24th SDM Conference;* AIAA Paper 83-0864, May 1983.

[21]Hsu, C. S., "On the Dynamic Stability of Elastic Bodies with Prescribed Initial Conditions," *Int'l J. Eng. Sci.,* Vol. 4, 1966, pp. 1-21.

[22]Hsu, C. S., "The Effects of Various Parameters on the Dynamic Stability of a Shallow Arch," *Journal of Applied Mechanics,* Vol. 34, No. 2, 1967, pp. 349-358.

[23]Lock, M. H., "The Snapping of a Shallow Sinusoidal Arch Under a Step Pressure Load," *AIAA Journal,* Vol. 4, July 1966, pp. 1249-1256.

[24]Simitses, G. J., "Suddenly-Loaded Structural Configurations," *J. of Eng. Mech.,* Div. ASCE, Vol. 110, No. 9, 1984, pp. 1320-1334.

[25]Walker, K. P., "Research and Development Programs for Nonlinear Structural Modeling with Advanced Time Temperature Dependent Constitutive Relations," NASA-CR 165533, 1981.

[26]Chan, K. S., Bodner, S. R., Walker, K. P., and Lindholm, U. S., "A Survey of Unified Constitutive Theories," NASA CR-23925, 1984.

[27]Riff, R., Carlson, R. L., and Simitses, G. J., "The Thermodynamically Consistent Constitutive Equations For Nonisothermal Large Strain, Elasto-Plastic, Creep Behavior," *AIAA Journal,* Vol. 25, Jan. 1987, pp. 114-122.

[28]Simitses, G. J., Carlson, R. L., and Riff, R., "Formulation of the Nonlinear Analysis of Shell-Like Structures, Subjected to Time-Dependent Mechanical and Thermal Loading," NASA Rept. 1986.

[29]Zienkiewicz, O. C. and Cheung, Y. K., *The Finite Element Method in Structural and Continuum Mechanics,* McGraw-Hill, New York, 1967.

[30]Oden, J. T., *Finite Elements of Nonlinear Continua,* McGraw-Hill, New York, 1972.

[31]Ramm, E. "Strategies for Tracing the Nonlinear Response Near Limit Points," *Proceedings Europe-U.S. Workshop on "Nonlinear Finite Element Analysis in Structural Mechanics,"* Springer-Verlag, New York, 1981.